EFFECT OF THE INHOMOGENEITY OF THE TEMPERATURE FIELD IN THE SCREEN COOLED BY EXHAUST GASES ON THE EFFICIENCY OF A CRYOSTAT

I. S. Zhitomirskii, A. L. Yastrzhembskii, and V. A. Volosyuk

UDC 536.21

A quantitative estimate is obtained for the effect of the inhomogeneity of the temperature field in the cooled screen on the loss of material stored in a cryostat.

When carrying out thermal calculations relating to cryostats the temperature field in the screens cooled by the vapor of the stored substance is usually regarded as uniform (ideal screen). This assumption is quite acceptable if the ratio of the longitudinal conductive conductivity of the screen to the transverse conductivity of the thermal insulation $(\lambda\delta/l)/\left(l\left(\frac{\lambda_{1x}}{\delta_1}+\frac{\lambda_{2x}}{\delta_2}\right)\right)$ is large. However, the use of cryostats in cosmic (space) research imposes strict limitations upon their weight, at the same time demanding an increase in the storage time, which leads to the use of large cryostats with thin screens. Under such conditions the loss of the material stored in the cryostat associated with external heat inflow may differ considerably from that calculated on the assumption of an ideal screen. In this paper we shall estimate these differences and determine their relationship to the constructional parameters of the cryostat.

Let us consider a cryogenic vessel of cylindrical shape containing a condensed gas. The vessel 1 (Fig. 1) is surrounded by two layers 2 and 3 of thermal insulation (TI) with a cooled screen 4 between them. The exhaust vapor of the stored substance (coolant) passes through tubes disposed in the screen, taking up some of the external heat inflow. It is assumed that the cooling tubes lie along the generators of the cylindrical screen, the vapor is distributed uniformly between the tubes, and the number of tubes is fairly large, so that the temperature field in the screen may be regarded as axisymmetrical. We assume heat transfer between the screen and the vapor to be ideal; the temperature of the vapor is equal to the temperature of that part of the screen with which it is in contact.

Let us consider the steady-state process and neglect temperature variation through the thickness of the screen. We shall only allow for heat inflow through the lateral surface of the cylindrical vessel.

Let us take an orthogonal coordinate system (x, y) in a plane passing through the axis of the cylinder such that the Oy axis coincides with the axis of the cylinder, while the Ox axis is directed along the radius (Fig. 2). Considering the smallness of δ_1 and δ_2 by comparison with r_1, we shall regard the surface areas of the inner vessel, the screen, and the outer casing as approximately equal, denoting their value $F = 2\pi r_1 l$.

Let us regard the TI as a continuous medium and describe heat transfer within it by way of certain effective thermal conductivities, so reducing the problem of determining the thermal fluxes in the insulation to a two-dimensional heat-conduction problem. We note that for laminated vacuum seals (LVS) the effective thermal conductivity along the layers of insulation is 2-3 orders of magnitude greater than the effective thermal conductivity across the layers [2]. Assuming that λ_{1x}, λ_{2x}, λ_{1y}, λ_{2y} are averaged over the thickness of each layer of insulation, we may regard them as being constant within the corresponding layer.

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 3, pp. 486-493, March, 1977. Original article submitted March 26, 1976.

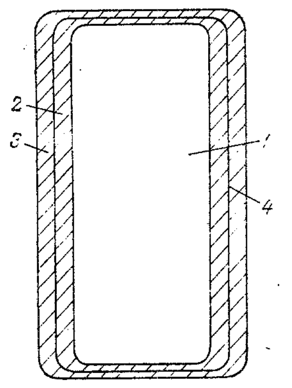

Fig. 1. Arrangement of cryostat.

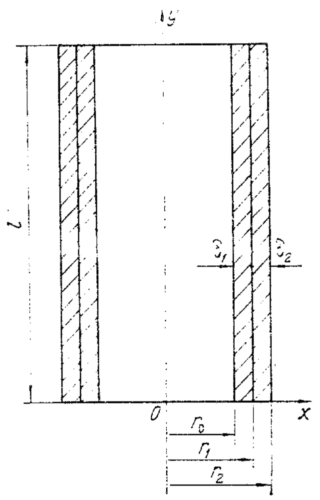

Fig. 2. Axial cross section of cryostat.

The temperature field T(x, y) of the inner and outer layers of insulation, respectively, satisfy the two-dimensional heat-conduction equations

$$\frac{\partial^2 T}{\partial x^2} + n_1^2 \frac{\partial^2 T}{\partial y^2} = 0, \; r_0 < x < r_1, \; 0 < y < l, \tag{1}$$

$$\frac{\partial^2 T}{\partial x^2} + n_2^2 \frac{\partial^2 T}{\partial y^2} = 0, \; r_1 < x < r_2, \; 0 < y < l, \tag{2}$$

where

$$n_1^2 = \frac{\lambda_{1y}}{\lambda_{1x}}, \; n_2^2 = \frac{\lambda_{2y}}{\lambda_{2x}}.$$

We consider that the thermal fluxes in the insulation at the boundaries y = 0 and y = l in the direction of the y axis are equal to zero. The temperature field of the inner layer of the TI then satisfies the boundary conditions

$$T(r_0, y) = T_0, \; T(r_1, y) = T_1(y), \; 0 < y < l, \qquad \left.\frac{\partial T}{\partial y}\right|_{y=0} = \left.\frac{\partial T}{\partial y}\right|_{y=l} = 0, \quad r_0 < x < r_1, \tag{3}$$

and that of the outer layer, the conditions

$$T(r_2, y) = T_2, \; T(r_1, y) = T_1(y), \; 0 < y < l, \qquad \left.\frac{\partial T}{\partial y}\right|_{y=0} = \left.\frac{\partial T}{\partial y}\right|_{y=l} = 0, \quad r_1 < x < r_2. \tag{4}$$

The temperature of the screen $T_1(y)$ satisfies the equation

$$\lambda A \frac{d^2 T_1}{dy^2} - G c_p \frac{dT_1}{dy} = -q_1(y), \tag{5}$$

where

$$q_1(y) = 2\pi r_1 \left(\lambda_{2x} \left.\frac{\partial T}{\partial x}\right|_{x=r_1+0} - \lambda_{1x} \left.\frac{\partial T}{\partial x}\right|_{x=r_1-0} \right).$$

The quantities λ and c_p are regarded as constant. At the instant of passing into the screen tube, the vapor has a temperature T_{init}, which is higher than T_0 and is determined from the thermal balance at the lower end section of the screen:

$$\lambda A \left.\frac{\partial T_1}{\partial y}\right|_{y=0} = G c_p (T_{init} - T_0). \tag{6}$$

This equation means that the thermal flux leaking from the end of the screen y = 0 is absorbed by the vapor before it passes into the cooling tubes. The screen temperature satisfies the boundary conditions

$$T_1(0) = T_{\text{init}}, \quad \left. \frac{dT_1}{dy} \right|_{y=l} = 0. \tag{7}$$

We shall call Eqs. (1)-(7) the fundamental problem.

Before analyzing this problem let us consider a cryostat with an ideal screen. Let the screen temperature T_1 be constant, while all the remaining assumptions of the fundamental problem remain valid. We consider that the temperature of the vapor leaving the screen is the same as the screen temperature. The rate of flow of the stored substance is

$$G_0 = q/r, \tag{8}$$

where $q = F(\lambda_{1x}/\delta_1)(T_1 - T_0)$ is the inflow of heat to the coolant. Let us write down the heat-balance equation for the screen

$$G_0 c_p (T_1 - T_0) = Q - q, \tag{9}$$

where $Q = F(\lambda_{2x}/\delta_2)(T_2 - T_1)$ is the thermal flux through the outer packet of the TI. Introducing the notation

$$B_0 = \frac{G_0 c_p}{F\left(\frac{\lambda_{1x}}{\delta_1} + \frac{\lambda_{2x}}{\delta_2}\right)}, \quad \mu = \frac{1}{1 + \frac{\lambda_{2x}\delta_1}{\lambda_{1x}\delta_2}},$$

from Eq. (9) we find

$$T_1 = \frac{1}{B_0 + 1}\left[(1 - \mu) T_2 + (B_0 + \mu) T_0\right].$$

Substituting T_1 into (8) we obtain an equation for determining B_0, and, of course, the flow rate G_0:

$$B_0(B_0 + 1) = K, \tag{10}$$

where

$$K = \frac{c_p}{r}\mu(1 - \mu)(T_2 - T_0). \tag{11}$$

We shall use Eq. (10) to compare the flow rates in the case of ideal and actual screens.

Let us return to the fundamental problem. Solving the problem (5)-(7), we obtain

$$T_1(y) = T_0 + \frac{1}{Gc_p}\int_0^l I(y, \xi)\, q_1(\xi)\, d\xi, \tag{12}$$

where

$$I(y, \xi) = \begin{cases} 1, & \xi \leqslant y, \\ \exp\left[\frac{Gc_p}{\lambda A}(y - \xi)\right], & \xi \geqslant y. \end{cases}$$

For the boundary problems (1), (3) and (2), (4) the Fourier method gives the following representation of the temperature field:

$$T(x, y) = T_0 + \sum_{k=1}^{\infty} \tau_k \frac{\operatorname{sh}\left[\frac{n_1 \pi k}{l}(x - r_0)\right]}{\operatorname{sh}\left(\frac{n_1 \pi k}{l}\delta_1\right)} \cos\frac{k\pi y}{l} + \frac{\tau_0}{\delta_1}(x - r_0) - T_0\frac{x - r_0}{\delta_1} \tag{13}$$

for $r_0 \leqslant x \leqslant r_1$,

$$T(x, y) = T_2 + \sum_{k=1}^{\infty} \tau_k \frac{\operatorname{sh}\left[\frac{n_2 \pi k}{l}(r_2 - x)\right]}{\operatorname{sh}\left(\frac{n_2 \pi k}{l}\delta_2\right)} \cos\frac{k\pi y}{l} + \frac{\tau_0}{\delta_2}(r_2 - x) - T_2\frac{r_2 - x}{\delta_2} \tag{14}$$

for $r_1 \leqslant x \leqslant r_2$. Here $\tau_0, \tau_1, \tau_2 \ldots$ are coefficients of the Fourier cosine-series expansion of the function $T_1(y)$. Making use of (12) we write these coefficients in the form

$$\tau_0 = \frac{1}{l}\int_0^l \left[T_0 - \frac{1}{Gc_p}\int_0^l I(y, \xi)\, q_1(\xi)\, d\xi\right] dy,$$

$$\tau_k = \frac{2}{l}\int_0^l \left[T_0 + \frac{1}{Gc_p}\int_0^l I(y, \xi)\, q_1(\xi)\, d\xi\right] \cos\frac{k\pi y}{l}\, dy, \quad k = 1, 2, \ldots,$$

from which, after expression $q_1(y)$ in terms of $T(x, y)$, we obtain an infinite system of equations for determining an infinite number of unknowns $\tau_0, \tau_1, \tau_2, \ldots$. The first of these equations is

$$B\tau_0 = BT_0 + b\Phi_{00} - \sum_{i=0}^{\infty} a_i \Phi_{i0}\tau_i, \tag{15}$$

where

$$B = \frac{Gc_p}{F\frac{\lambda_{1x}}{\delta_1} + F\frac{\lambda_{2x}}{\delta_2}}; \; b = \mu T_0 + (1-\mu)\, T_2;$$

$$a_0 = 1, \; a_i = \frac{\pi i}{l}\left[\mu\delta_1 n_1 \operatorname{cth}\left(\frac{n_1 \pi i}{l}\delta_1\right) + (1-\mu)\,\delta_2 n_2 \operatorname{cth}\left(\frac{n_2\pi i}{l}\delta_2\right)\right] \tag{16}$$

$$(\text{for } i = 1, 2, \ldots),$$

$$\Phi_{i0} = \frac{1}{l^2}\int_0^l \left(\int_0^l I(y, \xi)\cos\frac{i\pi\xi}{l}\, d\xi\right) dy \qquad (\text{for } i = 0, 1, 2, \ldots).$$

Each of the coefficients Φ_{i0} is expressed in terms of the complex

$$P = \frac{Gc_p}{\lambda A}\, l \tag{17}$$

[for example, $\Phi_{00} = 1/2 + (1/P) - (1/P^2) + (1/P^2)\exp(-P)$]. Let us express this complex in the form of a product $P = \varepsilon B$, where $\varepsilon = (l^2/\lambda\delta)[(\lambda_{1x}/\delta_1) + (\lambda_{2x}/\delta_2)]$ is the parameter defining the deviation from ideal conditions (imperfection parameter). Estimating ε for $l = 1$ m, $\delta_1 = \delta_2 = 0.05$ m, $\delta = 0.001$ m, $\lambda_{1x} = 10^{-5}$ W/m•deg, $\lambda_{2x} = 10^{-4}$ W/m•deg, $\lambda = 50$ W/m•deg, we obtain $\varepsilon \cong 0.044$. We note that $\varepsilon \to 0$ as $\delta\lambda$ increases. In order to obtain the unknown dependence of the rate of flow G of the stored substance on the parameters λ and δ let us consider the relationships between the dimensionless complexes B and ε which include G and $\delta\lambda$, respectively (an analogous method was used in [1]).

Let us determine the heat inflow q through the inner packet of insulation to the vessel:

$$q = \int_0^l 2\pi r_1 \lambda_{1x} \frac{\partial T}{\partial x}\bigg|_{x=r_1-0} dy = F\frac{\lambda_{1x}}{\delta_1}(\tau_0 - T_0).$$

Then

$$G = F\frac{\lambda_{1x}}{\delta_1}(\tau_0 - T_0)\frac{1}{r}, \tag{18}$$

whence

$$B\frac{r}{c_p\mu} = \tau_0(B, \varepsilon) - T_0, \text{ where } B = B(\varepsilon). \tag{19}$$

Let $\varepsilon \to 0$. Denoting B(0) by B_0 and $\tau_0(B_0, 0)$ by τ_0^0 and making use of (19), (15), and (17), we obtain an equation for determining B_0 coinciding with (10) for an ideal screen.

Now let $\varepsilon \neq 0$. After differentiating Eq. (15) with respect to ε and making ε tend toward zero, we obtain

$$B_0'\tau_0^0 + B_0\left[\left(\frac{\partial\tau_0}{\partial B}\right)_0 B_0' + \left(\frac{\partial\tau_0}{\partial\varepsilon}\right)_0\right] = B_0' T_0 + b(\Phi_{00}')_0 B_0 - \tau_0^0(\Phi_{00}')_0 B_0 - (\Phi_{00})_0\left[\left(\frac{\partial\tau_0}{\partial B}\right)_0 B_0' + \left(\frac{\partial\tau_0}{\partial\varepsilon}\right)_0\right]. \tag{20}$$

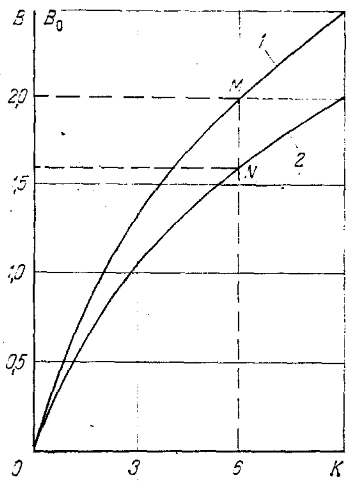

Fig. 3. Dependence of B and B_0 on K, respectively, for non-heat-conducting and ideal screens. The points M and N show the values of B and B_0 for a helium cryostat: 1) $B_0^2 + B_0 = K$; 2) (24).

From (19) and (17) we have

$$\left(\frac{\partial \tau_0}{\partial B}\right)_0 B_0' + \left(\frac{\partial \tau_0}{\partial \varepsilon}\right)_0 = B_0' \frac{r}{c_p}, \quad \tau_0^0 = B_0 \frac{r}{c_p} \frac{1}{\mu} + T_0,$$

$$(\Phi_{00}')_0 = -\frac{1}{6}, \quad (\Phi_{00})_0 = 1.$$

We now determine B_0' from Eq. (20):

$$B_0' = -\frac{\frac{1}{6} B_0 \left[-B_0 + \mu(1-\mu)\frac{c_p}{r}(T_2 - T_0)\right]}{2B_0 + 1},$$

whence allowing for (10) $B_0' = -{}^1/_6 B_0^3/(2B_0 + 1)$.

From the Taylor formula $B - B_0 = B_0'\varepsilon + 0(\varepsilon)$ we obtain the unknown dependence of B on ε

$$B - B_0 = -\frac{\frac{1}{6} B_0^3}{2B_0 + 1}\varepsilon + 0(\varepsilon). \tag{21}$$

This relationship is suitable for small values of the imperfection parameter ε such as occur in practical situations (remember the foregoing value of ε = 0.044). Using the earlier initial data, for a cryostat containing helium ($c_p/r = 0.25$) we find that the difference in rates of flow determined by Eq. (21) for arrangements with a constant and variable screen temperature is approximately 0.6%. Since the right-hand side of Eq. (21) is negative, we have $G < G_0$, i.e., the flow of the stored substance is greater in the case of an ideal screen than in the practical case. This difference is greater, the greater the value of the parameter ε.

It is thus interesting to consider the arrangement of a cryostat with a non-heat-conducting screen ($\varepsilon \to \infty$). We shall also put $\lambda_{1y} = \lambda_{2y} = 0$ (no thermal conductivity along the layers of insulation). All the remaining assumptions of the fundamental problem are retained. Under these conditions the temperature of the screen T_1 satisfies the equation

$$Gc_p \frac{dT_1}{dy} = q_1(y), \tag{22}$$

where

$$q_1(y) = 2\pi r_1 \left[\lambda_{2x} \frac{T_2 - T_1(y)}{\delta_2} - \lambda_{1x} \frac{T_1(y) - T_0}{\delta_1}\right],$$

and the boundary condition

$$T_1(0) = T_0. \tag{23}$$

Solving the problem (22)-(23, we obtain

$$T_1(y) = T_0 \exp\left(-\frac{y}{Bl}\right) + b\left[1 - \exp\left(-\frac{y}{Bl}\right)\right].$$

After determining the Fourier coefficient

$$\tau_0 = \frac{1}{l}\int_0^l T_1(y)\,dy = b + B(T_0 - b)\left[1 - \exp\left(-\frac{1}{B}\right)\right],$$

we use Eq. (18) to find the rate of flow

$$G = \lambda_{1x}\frac{F}{\delta_1}\left\{b + B(T_0 - b)\left[1 - \exp\left(-\frac{1}{B}\right)\right] - T_0\right\}\frac{1}{r}.$$

Using (11) and (16) we obtain an equation for determining the complex B

$$\frac{B}{1 - B\left[1 - \exp\left(-\frac{1}{B}\right)\right]} = K. \qquad (24)$$

We note that the right-hand sides of Eqs. (10) and (24) are equal. We see from Fig. 3 that $B < B_0$. The relative change in the rate of flow on passing from the ideal to the non-heat-conducting screen (original data as before) amounts to 20% for helium.

Thus the reduction in rate of flow on passing from an ideal screen to a real one (having a finite thermal conductivity) is fairly slight. However, the transition from an ideal to a non-heat-conducting screen (in the absence of heat flows along the insulation) yields a considerable reduction in flow rate, which indicates the undesirability of using non-heat-conducting cooled screens in cryostats.

NOTATION

TI, thermal insulation; LVS, laminated vacuum seal; r_0, r_1, r_2, radii of the cryogenic vessel, screen, and outer casing; δ_1, δ_2, thicknesses of the inner and outer packets of insulation; δ, thickness of screen; l, length of screen; F, surface area of screen; A, area of screen cross section perpendicular to the axis of the vessel; λ_{1x}, λ_{2x}, effective thermal conductivities in a direction perpendicular to the layers of insulation for the inner and outer packets of the LVS; λ_{1y}, λ_{2y}, effective thermal conductivities along the layers for the inner and outer packets of the LVS; $n_1^2 = \lambda_{1y}/\lambda_{1x}$; $n_2^2 = \lambda_{2y}/\lambda_{2x}$; λ, thermal conductivity of the screen material; c_p, specific heat of the vapor of the stored substance at constant pressure; r, latent heat of vaporization of the cryogenic substance; T_0, temperature inside the cryogenic vessel; T_2, ambient temperature; T(x, y), temperature field in the insulation; $T_1(y)$, screen temperature; q(y), Q(y), q, Q, thermal flows through the inner and outer packets of insulation, value referred to unit length of the generator and total value, respectively; $q_1 = Q - q$; G_0, flow rate of stored substance for a cryostat with an ideal screen; G, flow rate of substance for a cryostat with a nonuniform temperature field in the screen; $\varepsilon = (l^2/\lambda\delta)[(\lambda_{1x}/\delta_1) + (\lambda_{2x}/\delta_2)]$, imperfection parameter; $B_0 = G_0c_p/[F(\lambda_{1x}/\delta_1) + F(\lambda_{2x}/\delta_2)]$, $B = Gc_p/[F(\lambda_{1x}/\delta_1) + F(\lambda_{2x}/\delta_2)]$, $\mu = 1/[1 + (\lambda_{2x}\delta_1/\lambda_{1x}\delta_2)]$, $K = (c_p/r)\mu(1 - \mu)(T_2 - T_0)$, $b = \mu T_0 + (1 - \mu)T_2$, dimensionless complexes.

LITERATURE CITED

1. B. I. Verkin and I. S. Zhitomirskii, Izv. Akad. Nauk SSSR, Énerget. Transport, No. 5 (1974).
2. D. P. Lebedev, E. K. Zlobin, and V. V. Alekseev, Inzh.-Fiz. Zh., 21, No. 5 (1971).